Limit Laws of The Coefficients of Polynomials with Only Unit Roots

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Abstract

We consider sequences of random variables whose probability generating functions have only roots on the unit circle, which has only been sporadically studied in the literature. A necessary and sufficient condition is given of the asymptotic normality, which differs from the situation when all roots are real. We also derive a representation theorem for all possible limit laws and apply our results to many concrete examples in the literature, ranging from combinatorial structures to numerical analysis, and from probability to analysis of algorithms.